GRADE 12 DIPLOMA EXAMINATION

Mathematics 30

January 1985

Alberta
EDUCATION

GRADE 12 DIPLOMA EXAMINATION
MATHEMATICS 30

DESCRIPTION

Time: 2½ hours

Total possible marks: 65

This is a **CLOSED-BOOK** examination consisting of two parts:

PART A: 52 multiple-choice questions each with a value of 1 mark.

PART B: Five written-response questions for a total of 13 marks.

A mathematics data booklet is provided for your reference. Approved calculators may be used.

GENERAL INSTRUCTIONS

Fill in the information on the answer sheet as directed by the examiner.

For multiple-choice questions, read each carefully and decide which of the choices BEST completes the statement or answers the question. Locate that question number on the answer sheet and fill in the space that corresponds to your choice. USE AN HB PENCIL ONLY.

Example **Answer Sheet**

This examination is for the subject area of

A	B	C	D
①	②	③	●

A. Chemistry
B. Biology
C. Physics
D. Mathematics

If you wish to change an answer, please erase your first mark completely.

For written-reponse questions, read each carefully, show all your calculations, and write your answer in the space provided in the examination booklet.

NOTE: The perforated pages at the back of this booklet may be torn out and used for your rough work.

DO NOT FOLD EITHER THE ANSWER SHEET OR THE EXAMINATION BOOKLET

The presiding examiner will collect the answer sheet and examination booklet for transmission to Alberta Education.

JANUARY 1985

PART A

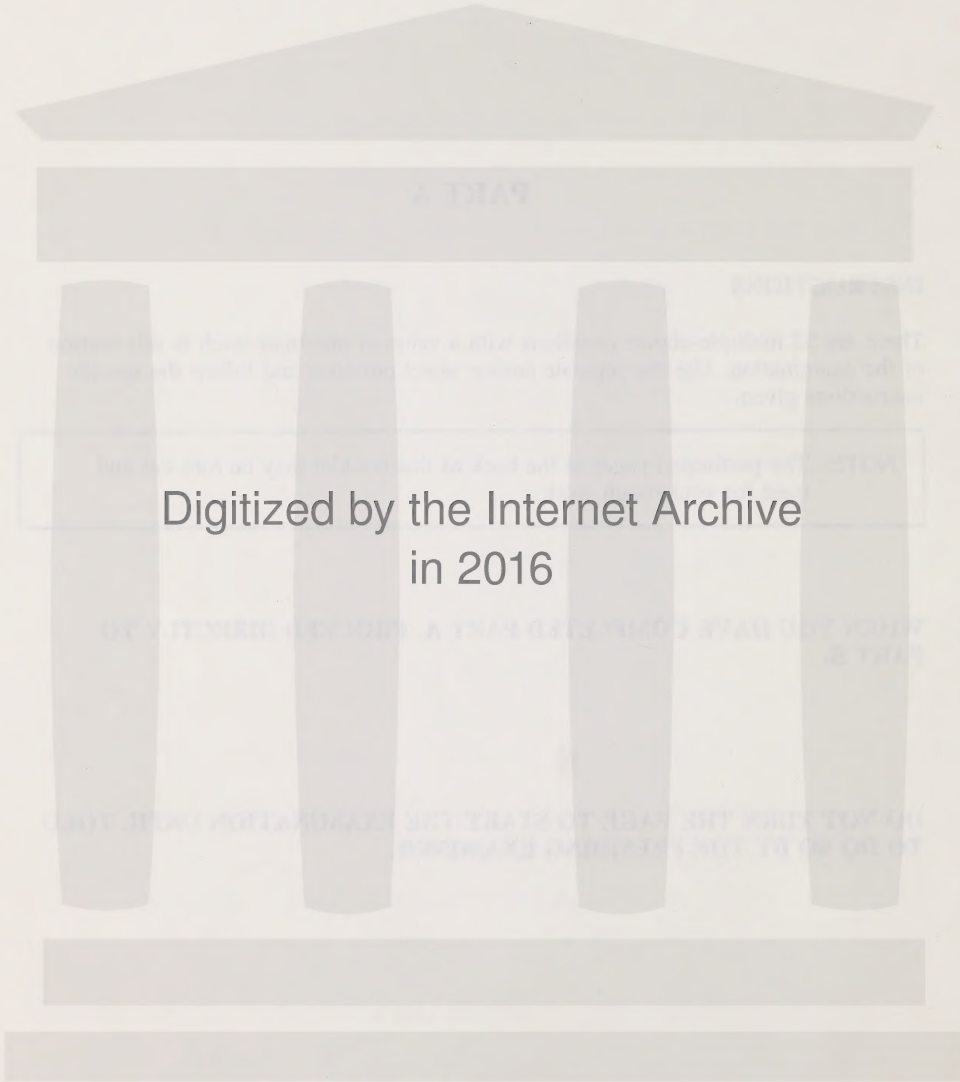
INSTRUCTIONS

There are 52 multiple-choice questions with a value of one mark each in this section of the examination. Use the separate answer sheet provided and follow the specific instructions given.

NOTE: The perforated pages at the back of this booklet may be torn out and used for your rough work.

WHEN YOU HAVE COMPLETED PART A, PROCEED DIRECTLY TO PART B.

DO NOT TURN THE PAGE TO START THE EXAMINATION UNTIL TOLD TO DO SO BY THE PRESIDING EXAMINER.



1. From the top of a 65 m cliff, the angle of depression of a boat in the harbor below is 54°. How far (to the nearest metre) is the boat from the base of the cliff?

A. 111 m
B. 80 m
C. 53 m
D. 47 m

2. On a unit circle a path of $\frac{3\pi}{4}$ radians would have the same initial and terminal points as a path of

A. $\frac{29\pi}{4}$ radians

B. $-\frac{29\pi}{4}$ radians

C. $\frac{7\pi}{4}$ radians

D. $-\frac{11\pi}{4}$ radians

3. If $\cos\theta = -\sin\theta$, then $\sin^2\theta$ is

A. $-\frac{\sqrt{2}}{2}$

B. $\frac{1}{2}$

C. $\frac{1}{4}$

D. $\frac{\sqrt{2}}{2}$

4. All the solutions of $4\cos^2\theta = 3, 0 \leq \theta < 2\pi$ are

A. $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

B. $\frac{\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{6}$

C. $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

D. $\frac{\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12}$

5. If $\cos\theta = -\frac{8}{17}$ and θ is a second quadrant angle, then $\tan\theta$ equals

A. $-\frac{15}{17}$

B. $-\frac{17}{15}$

C. $-\frac{8}{15}$

D. $-\frac{15}{8}$

6. If $\theta \neq n\pi, n \in I$, $\csc^2\theta - \cot^2\theta - \sin^2\theta$ is equal to

A. $1 + \cot^2\theta$

B. $\sec^2\theta$

C. $1 + \sec^2\theta$

D. $\cos^2\theta$

7. If $\sin A \neq 0$, then $\frac{\sin(2A)}{\sin A}$ is equivalent to

A. $\sin A$
B. $\cos A$
C. $2\sin A$
D. $2\cos A$

8. For the function $y = 3\sin(2\theta)$, the period is

A. π
B. 2π
C. 3π
D. 4π

9. On a unit circle, a central angle of 160° corresponds to a path length of

A. $\frac{9\pi}{8}$

B. $\frac{8\pi}{9}$

C. $\frac{7\pi}{8}$

D. $\frac{7\pi}{9}$

10. The exact value of $\sin 240° + \cos 690°$ is

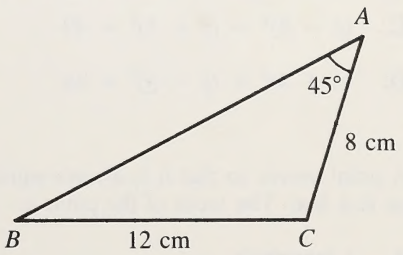
A. -1

B. $-\frac{\sqrt{3}}{2}$

C. 0

D. $\frac{1}{2}$

11. In the triangle shown at the right, the measure of $\angle C$ to the nearest degree is



A. 107°

B. 100°

C. 97°

D. 28°

12. Town A is 60 km from town C, and town B is 20 km from town C and 50 km from town A. The measure of $\angle BAC$ to the nearest degree is

A. 18°
B. 19°
C. 22°
D. 35°

13. If a regular hexagon has sides of 10 cm, then the area is

A. 60 cm^2

B. 100 cm^2

C. $60\sqrt{3} \text{ cm}^2$

D. $150\sqrt{3} \text{ cm}^2$

14. Point $P(x, y)$ moves such that the line segments joining P to $(-3, 0)$ and to $(5, 0)$ are perpendicular. The equation of the locus of P is

A. $(x - 4)^2 + y^2 = 31$

B. $(x + 4)^2 + y^2 = 31$

C. $(x - 1)^2 + y^2 = 16$

D. $(x - 1)^2 + y^2 = -14$

15. A circle with centre at the point $(4, -5)$ and radius 7 units is defined by the equation

A. $(x + 4)^2 + (y - 5)^2 = 7$

B. $(x - 4)^2 + (y + 5)^2 = 7$

C. $(x - 4)^2 + (y + 5)^2 = 49$

D. $(x + 4)^2 + (y - 5)^2 = 49$

16. A point moves so that it is always equidistant from a given line and a given point not on that line. The locus of the point is

A. a hyperbola
B. a circle
C. an ellipse
D. a parabola

17. The directrix of $x^2 = -12y$ is

A. $y = -3$
B. $y = 3$
C. $x = -3$
D. $x = 3$

18. The parabola whose vertex is at (2, 4) and whose focus is at (2, 6) is defined by the equation

A. $(y - 4)^2 = 8(x - 2)$

B. $(y - 4)^2 = -8(x - 2)$

C. $(x - 2)^2 = -8(y - 4)$

D. $(x - 2)^2 = 8(y - 4)$

19. If a man throws a ball in the path of a parabolic arch defined by $x^2 = -16y$, and the ball strikes the ground 56 m from the man, then the maximum height reached by the ball is

A. $3\frac{1}{2}$ m

B. $8\sqrt{14}$ m

C. 49 m

D. 196 m

20. If the equation of an ellipse is $25x^2 + 9y^2 = 225$, then the length of the major axis is

A. 10
B. 9
C. 6
D. 5

21. The equation of the ellipse centred at the origin, with minor axis of 10 units and one focus at (0, 2), is

A. $\frac{x^2}{21} + \frac{y^2}{25} = 1$

B. $\frac{x^2}{25} + \frac{y^2}{29} = 1$

C. $\frac{x^2}{25} + \frac{y^2}{21} = 1$

D. $\frac{x^2}{29} + \frac{y^2}{25} = 1$

22. The graph of $\frac{x^2}{10} - \frac{y^2}{3} = 1$ is

A. a parabola
B. a circle
C. a hyperbola
D. an ellipse

23. The equations of the asymptotes of the hyperbola $9x^2 - 16y^2 = 144$ are

A. $y = \pm \frac{16}{5}x$

B. $y = \pm \frac{4}{3}x$

C. $y = \pm \frac{3}{4}x$

D. $y = \pm \frac{5}{16}x$

24. A point (x, y) moves so that the difference between its distances from $(-6, 0)$ and $(6, 0)$ is always 8 units. One possible value of y when x equals 6 is

A. 65

B. 25

C. $\sqrt{65}$

D. 5

25. Assuming the sequence $-1, 3, 7, \cdots$ is arithmetic, the 19th term is

A. 71
B. 73
C. 75
D. 77

26. A debt of \$3600 is to be repaid over a period of 3 years by making payments of \$100 at the end of each month plus the interest at the rate of 2% per month on the unpaid balance. How much interest will have been paid by the time the debt is repaid?

A. \$1332
B. \$1296
C. \$1260
D. \$1240

27. If in a geometric sequence the 6th term is 6 and the 9th term is 162, then the 3rd term is

A. $\frac{2}{81}$

B. $\frac{2}{9}$

C. 3

D. 27

28. $\sum_{k=1}^{20} (4k - 6)$ is equal to

A. 720
B. 210
C. 204
D. 90

29. $\lim_{n \to \infty} \left(\frac{1 - 2n + 3n^2}{3 + 3n - 5n^2} \right)$ is

A. not defined

B. $\frac{1}{3}$

C. 0

D. $-\frac{3}{5}$

30. An example of a convergent infinite sequence is

A. $2, 4, 8, \cdots, (2^n), \cdots$

B. $2, 3, 4, \cdots, (n + 1), \cdots$

C. $2, 1, \frac{1}{2}, \cdots, \left(\frac{4}{2^n} \right), \cdots$

D. $2, 1, 0, \cdots, (3 - n), \cdots$

31. The limit of the sequence $-3, 3, -3, \cdots, 3(-1)^n, \cdots$ is

A. non-existent
B. 0
C. -3
D. 3

32. The sum of the series $20 + \left(5 - \frac{5}{2} + \frac{5}{4} - \frac{5}{8} + \cdots + 5(-\frac{1}{2})^{n-1} + \cdots\right)$ is

A. $3\frac{1}{3}$

B. 10

C. $23\frac{1}{3}$

D. 30

33. In a weight-loss program, a 150 kg man loses 8 kg in the first 10 weeks, 6 kg in the next 10 weeks, and 4.5 kg in the following 10 weeks. If this geometric sequence of losses continues, his weight will approach

A. 131.5 kg
B. 118 kg
C. 64 kg
D. 32 kg

34. The range of the data shown at the right is

15	18	26	23	38
14	31	23	18	35
32	23	19	20	31
21	16	34	30	17

A. 52
B. 38
C. 24
D. 23

35. In a normal distribution, the data are distributed so that 98.76% of the data are within

A. 1.5 standard deviations of the mean
B. 2.5 standard deviations of the mean
C. 3.5 standard deviations of the mean
D. 4.5 standard deviations of the mean

36. From previous observations, police know that the speeds of cars in a 70 km/h zone are normally distributed about a mean of 73.5 km/h, with a standard deviation of 3.5 km/h. A radar speed trap is set up in a 70 km/h zone and the speeds of 200 cars are recorded. If the police must allow 10% of the posted limit as a margin for error, the number of cars stopped for speeding is

A. 168
B. 136
C. 68
D. 32

37. A generator manufacturer determines the mean life of his generators to be 6 years, with a standard deviation of 2 years. If he guarantees his generators for 3 years, the percentage of generators that he will have to replace is

A. 6.7%
B. 33%
C. 43.3%
D. 50%

38. The time required for an athlete to complete an obstacle course is normally distributed with a mean of 20 min and a standard deviation of 0.5 min. The percentage of athletes with times less than 20.5 min is

A. 98%
B. 84%
C. 50%
D. 16%

39. On a university entrance exam, the mean of all the scores was 55, with a standard deviation of 5. If John's score on the test was 47, what was his z-score?

A. -0.42
B. 0.42
C. -1.60
D. 1.60

40. The "life" of a Brand X vacuum cleaner is normally distributed with a standard deviation of 24 months. The manufacturer guarantees the vacuum cleaner for 4 years. If the probability that a vacuum cleaner is returned under this guarantee is 0.05, then the mean life of the vacuum is

A. 87 months
B. 60 months
C. 51 months
D. 45 months

41. From the table at the right, the estimated probability that a tube will not last longer than 650.5 h is

A. 0.30
B. 0.38
C. 0.62
D. 0.70

Lifetime of 320 Radio Tubes	
Lifetime (h)	Frequency
400.5 - 475.5	25
475.5 - 550.5	30
550.5 - 625.5	40
625.5 - 700.5	81
700.5 - 775.5	70
775.5 - 850.5	55
850.5 - 925.5	19

42. Solve for x: $8^{3x-4} = 16^{4x+1}$

A. $-\frac{16}{7}$

B. $\frac{16}{7}$

C. $-\frac{6}{5}$

D. $\frac{6}{5}$

43. Which of the following is equivalent to the equation $\log(F) = 2\log(V) - \log(r)$?

A. $F = 2V - r$

B. $F = \frac{2V}{r}$

C. $F = V^2 - r$

D. $F = \frac{V^2}{r}$

44. The logarithmic form of $81^{\frac{1}{2}} = 9$ is

A. $\log_{81}(9) = \frac{1}{2}$

B. $\log_{9}(81) = \frac{1}{2}$

C. $\log_{\frac{1}{2}}(9) = 81$

D. $\log_{9}\left(\frac{1}{2}\right) = 81$

45. If $y = 5^x$, $x \in R$, then the inverse function is

A. $y = \log_x(5)$

B. $y = \log_5(x)$

C. $y = \log(5)$

D. $y = \log(x)$

46. The value of x in the equation $(4.5)^{x-1} = 26$ is

A. 1.17
B. 3.17
C. 2.97
D. 0.97

47. $\frac{\log(r)}{t} + \frac{\log(d)}{t}$ is equivalent to

A. $\log(rd)^t$

B. $\log \sqrt[t]{rd}$

C. $\log \sqrt[t]{(r + d)}$

D. $\log(r + d)^t$

48. The Richter scale determines the magnitude of an earthquake according to the formula $M = \log_{10}(I)$, where M is the magnitude of the earthquake and I is the intensity of the earthquake. An earthquake measuring 4.2 on the Richter scale hits the Fiji Islands, and another earthquake measuring 6.3 on the scale hits San Francisco. How many times more intense is the San Francisco earthquake than the one in Fiji?

A. 126
B. 58
C. 31.6
D. 26.5

49. For what value of m is $x^3 + 4x^2 - 5m$ divisible by $x - 1$?

A. 5
B. 1
C. -1
D. -5

50. The coefficient of x^3 in a third-degree polynomial $P(x)$ is 1. If $P(-1) = P(-2) = P(3) = 0$, then $P(x)$ is equal to

A. $x^3 + 6x^2 + 11x + 6$

B. $x^3 - 6x^2 - 7x - 6$

C. $x^3 - 7x + 6$

D. $x^3 - 7x - 6$

51. The y-intercept of the graph of a third-degree polynomial $P(x)$ is 6. If the x-intercepts are $3, -\frac{1}{2}$, and $\frac{2}{3}$, then $P(x)$ equals

A. $6x^3 + 25x^2 + 21x + 6$

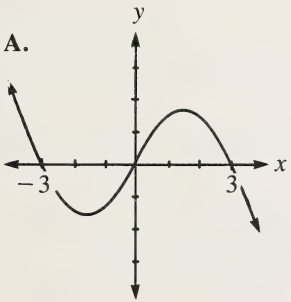
B. $6x^3 + 25x^2 + x + 6$

C. $6x^3 - 19x^2 - x + 6$

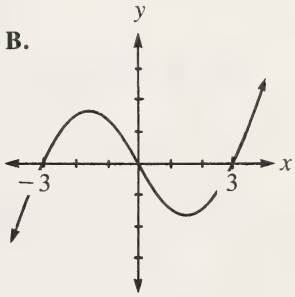
D. $6x^3 - 19x^2 + x + 6$

52. The sketch that best represents the graph of $y = -3x^3 + 27x$ is

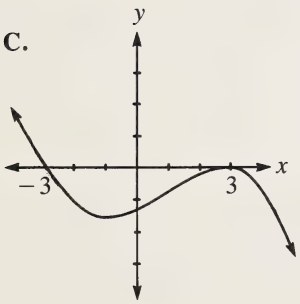
A.



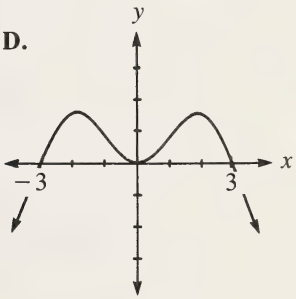
B.



C.



D.



YOU HAVE NOW COMPLETED THE MULTIPLE-CHOICE SECTION OF THE EXAMINATION. PLEASE PROCEED TO THE NEXT PAGE AND ANSWER THE WRITTEN-RESPONSE QUESTIONS IN PART B.

PART B

INSTRUCTIONS

Please write your answers in the examination booklet as neatly as possible.

Show all pertinent calculations and formulas, and give your answers to the correct number of significant figures.

NOTE: The perforated pages at the back of this booklet may be torn out and used for your rough work.

TOTAL MARKS: 13

START PART B IMMEDIATELY

(2 marks) **1.** Two aircraft leave the Edmonton airport at the same time, one flying at 700 km/h and the other at 850 km/h. After 2.5 h they are 1900 km apart. Find the angle between their flight paths to the nearest degree.

(3 marks) **2.** An elliptical dinner plate just fits on a rectangular placemat 30 cm long and 24 cm wide. A knife laid on the plate just fits between the two foci of the ellipse. Find the length of the knife.

(3 marks) **3.** A knitting pattern has an initial row of 7 stitches. Each successive row increases by 3 stitches until a row of exactly 64 stitches is reached. After that, each successive row decreases by 4 stitches. In which row(s) can exactly 44 stitches be found?

(2 marks) **4.** If a set of data follows a normal distribution with a mean of 15 and a standard deviation of 2.4, between what two values will 95% of the data be found?

(3 marks) **5.** Express the polynomial $x^4 - 2x^3 - 5x^2 + 4x + 6$ as a product of first-degree factors.

YOU HAVE NOW COMPLETED THE EXAMINATION. IF YOU HAVE TIME, YOU MAY WISH TO GO BACK AND CHECK YOUR ANSWERS.

TEAR-OUT SHEET

(NO MARKS WILL BE GIVEN FOR WORK DONE ON THIS PAGE)

FOLD AND TEAR ALONG PERFORATION

(NO MARKS WILL BE GIVEN FOR WORK DONE ON THIS PAGE)

TEAR-OUT SHEET

(NO MARKS WILL BE GIVEN FOR WORK DONE ON THIS PAGE)

FOLD AND TEAR ALONG PERFORATION

TEAR-OUT SHEET

(NO MARKS WILL BE GIVEN FOR WORK DONE ON THIS PAGE)

FOLD AND TEAR ALONG PERFORATION

FOR DEPARTMENT USE ONLY

M1 M2

FOR DEPARTMENT USE ONLY

MATHEMATICS 30

(LAST NAME) (FIRST NAME)

NAME:

Y M D

DATE OF BIRTH: SEX:

SCHOOL CODE: SCHOOL:

PERMANENT MAILING ADDRESS:

SIGNATURE:

FOR DEPARTMENT USE ONLY

MATHEMATICS 30